

The Gribov Horizon and Ghost Interactions in Euclidean Gauge Theories

Hirohumi Sawayanagi¹

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The effect of the Gribov horizon in Euclidean $SU(2)$ gauge theory is studied. Gauge fields on the Gribov horizon yield zero modes of ghosts and anti-ghosts. We show these zero modes can produce additional ghost interactions, and the Landau gauge changes to a nonlinear gauge effectively. In the infrared limit, however, the Landau gauge is recovered, and ghost zero modes may appear again. We show ghost condensation happens in the nonlinear gauge, and the zero mode repetition is avoided.
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1 Introduction

A perturbative calculation in gauge theories requires gauge fixing. However, in non-Abelian gauge theories, there is a problem of gauge copies [1]. Gribov showed that gauge-equivalent copies exist in the Landau gauge

$$\partial_\mu A_\mu = 0. \quad (1.1)$$

In the Coulomb gauge, it was shown that almost all gauge transformations are responsible for gauge fixing degeneracies [2]. If gauge copies are connected by an infinitesimal gauge transformation with a gauge parameter $\varepsilon(x)$, (1.1) gives $\partial_\mu D_\mu \varepsilon(x) = 0$. That is, the Faddeev-Popov (FP) operator $-\partial_\mu D_\mu$ has zero eigenvalues. The boundary that the lowest eigenvalue of the FP operator equals zero is called the (first) Gribov horizon $\partial\Omega$. The region inside $\partial\Omega$, where eigenvalues of $-\partial_\mu D_\mu$ are positive, is called the Gribov region Ω . In general, gauge copies may exist outside of Ω [1] and on the horizon [3].

There are some ideas to solve the problem. One of them is to restrict a functional integral in the Gribov region Ω [1, 4]. (Strictly speaking, there may be some copies in Ω . Hence more restricted region in Ω , that is called a fundamental modular region (FMR) Λ , is considered [5].) Another idea is to sum over all gauge copies [6, 7]. For a solvable gauge model, it was shown that correct results are obtained by collecting all gauge copies [8, 9].

The Gribov horizon yield some effects. In the first approach, the horizon perturbs gluons into shadow particles [4, 10]. Even if the region is restricted to the FMR Λ , there are points that the boundary of Λ touches the horizon $\partial\Omega$ [5]. These points give the singularity of the operator $1/\partial_\mu D_\mu$. As a result, the color Coulomb potential is enhanced and the confinement might be shown [11]. In the second approach, gauge configurations on the Gribov horizon contribute in general, and the FP operator has zero modes. These zero modes can cause a trouble in proving the gauge equivalence [12]. Thus physical effects of the horizon $\partial\Omega$ are worth studying.

In this paper, we study the effect of these zero modes. In the next section, we show that a pair of zero modes in the Landau gauge can yield additional ghost interactions. If we require the BRS invariance, an effective Lagrangian becomes a Lagrangian in a nonlinear gauge. In §3, the gauge $\partial_\mu A_\mu \neq 0$ is considered. If there is a pair of zero modes, the nonlinear gauge is realized as well. We also show that the partition function does not vanish even if the FP operator yields a single zero mode. In §4, the effect of a single zero mode is discussed in the Landau gauge. In the low energy region, ghost condensation appears in the nonlinear gauge. The effect of the zero modes under the condensation is discussed in §5. §6 is devoted to summary. In Appendix A, examples of zero modes in the Coulomb gauge are given in three

dimensional space-time. In Appendix B, the effective Lagrangian in §2 is derived by the use of a source term. The nonlinear gauge has two gauge parameters. Renormalization group equations for these parameters are presented in Appendix C. In Appendix D, symmetries in the nonlinear gauge are discussed.

2 Effect of ghost zero modes in the Landau gauge

We consider the $SU(2)$ gauge theory with structure constants f^{ABC} . Using the notations

$$F \cdot G = F^A G^A, \quad (F \times)^{AB} = f^{ACB} F^C, \quad (F \times G)^A = f^{ABC} F^B G^C, \quad A = 1, 2, 3,$$

a partition function in the Landau gauge is $Z_L = Z_{\alpha=0}$ with

$$Z_\alpha = \int D\mu e^{-\int dx (\mathcal{L}_{\text{inv}} + \mathcal{L}_\alpha)}, \quad D\mu = DA_\mu DB Dc D\bar{c}, \quad (2.1)$$

$$\mathcal{L}_{\text{inv}} = \frac{1}{4} F_{\mu\nu}^2, \quad \mathcal{L}_\alpha = B \cdot \partial_\mu A_\mu - \frac{\alpha}{2} B^2 + i\bar{c} \cdot \partial_\mu D_\mu c, \quad (2.2)$$

where $i\bar{c} \cdot \partial_\mu D_\mu c = i\bar{c}^A \partial_\mu (\partial_\mu + gA_\mu \times)^{AB} c^B$. The gauge condition (1.1) leads to the relations

$$\partial_\mu D_\mu = D_\mu \partial_\mu, \quad \int dx i\bar{c} \cdot \partial_\mu D_\mu c = \int dx i(\partial_\mu D_\mu \bar{c}) \cdot c. \quad (2.3)$$

Namely, $\partial_\mu D_\mu$ is hermitian, and its eigenvalues are real.

The eigenfunction u_n with the eigenvalue λ_n satisfies

$$-\partial_\mu D_\mu u_n(x) = \lambda_n u_n(x). \quad (2.4)$$

When A_μ is on the first Gribov horizon, the lowest eigenvalue is $\lambda_0 = 0$ and $u_0(x)$ is a zero mode. If we can make $u_0(x)$ complex, as (2.4) leads to

$$-\partial_\mu D_\mu u_n^*(x) = \lambda_n u_n^*(x), \quad (2.5)$$

$u_0^*(x)$ is also a zero mode. We assume a pair of zero modes $(u_0(x), u_0^*(x))$ exists. Some examples of a zero-mode pair $(u_0(x), u_0^*(x))$ are presented in Appendix A. If u_0 is real, it may be a single zero mode. An example of such a zero mode is given in Appendix A, and its effect is discussed in §4.

Now we expand the ghost c as ¹

$$c(x) = \xi u_0(x) + \xi^\dagger u_0^*(x) + \cdots, \quad (2.6)$$

where ξ and ξ^\dagger are independent Grassmann variables. Other modes, i.e. nonzero modes and a single zero mode, are not written explicitly. In the same way, the property (2.3) implies

¹ We assume that eigenfunctions of the FP operator form an orthonormal complete set. Strictly speaking, to ensure it, spaces and/or configurations of A_μ must be restricted. We emphasize what is important here is that c contains $\xi u_0, \xi^\dagger u_0^*$ and \bar{c} contains $\bar{\xi} u_0, \bar{\xi}^\dagger u_0^*$.

that the expansion

$$\bar{c}(x) = \bar{\xi} u_0(x) + \bar{\xi}^\dagger u_0^*(x) + \dots \quad (2.7)$$

holds. We note, if there are some pairs of zero modes $(u_0^j(x), u_0^{j*}(x))$ ($j = 1, 2, \dots$), $\xi u_0(x) + \xi^\dagger u_0^*(x)$ and $\bar{\xi} u_0(x) + \bar{\xi}^\dagger u_0^*(x)$ are replaced by $\sum_j [\xi_j u_0^j(x) + \xi_j^\dagger u_0^{j*}(x)]$ and $\sum_j [\bar{\xi}_j u_0^j(x) + \bar{\xi}_j^\dagger u_0^{j*}(x)]$, respectively. However the discussion below is also applicable.

Eqs.(2.4) and (2.5) imply that the Lagrangian $\int dx i\bar{c} \cdot \partial_\mu D_\mu c$ does not contain the Grassmann variables $\xi, \xi^\dagger, \bar{\xi}$ and $\bar{\xi}^\dagger$. However the measures Dc and $D\bar{c}$ contain $d\xi d\xi^\dagger$ and $d\bar{\xi} d\bar{\xi}^\dagger$, respectively. Since a Grassmann variable ζ satisfies

$$\int d\zeta \zeta^n = \begin{cases} 1 & (n = 1) \\ 0 & (n = 0, 2, 3, \dots) \end{cases} \quad (2.8)$$

the partition function vanishes:

$$\int Dc D\bar{c} e^{-\int dx \mathcal{L}_\alpha} = 0.$$

We know that fermions in an instanton background have zero modes. These zero modes yield the additional interaction of fermions [13, 14]. Likewise, the above ghost zero modes may produce additional ghost interactions, because

$$\int Dc D\bar{c} \xi \xi^\dagger \bar{\xi} \bar{\xi}^\dagger e^{-\int dx \mathcal{L}_\alpha} \neq 0. \quad (2.9)$$

From (2.6) and (2.7), we obtain

$$c^A c^B \bar{c}^C \bar{c}^D = \Psi^{ABCD} \xi \xi^\dagger \bar{\xi} \bar{\xi}^\dagger + \dots,$$

where $\Psi^{ABCD} = u_0^A u_0^B u_0^{*C} u_0^{*D}$, and terms denoted by \dots lack some or all of $\xi, \xi^\dagger, \bar{\xi}$ and $\bar{\xi}^\dagger$. Therefore (2.9) leads to

$$\int Dc D\bar{c} \sigma^{[AB][CD]} \Psi^{ABCD} \xi \xi^\dagger \bar{\xi} \bar{\xi}^\dagger e^{-\int dx \mathcal{L}_\alpha} = \int Dc D\bar{c} \sigma^{[AB][CD]} c^A c^B \bar{c}^C \bar{c}^D e^{-\int dx \mathcal{L}_\alpha}, \quad (2.10)$$

where $\sigma^{[AB][CD]}$ is antisymmetric with respect to A and B , and C and D as well. Thus ghost zero modes produce effective ghost interactions.

Now we determine $\sigma^{[AB][CD]}$, and construct effective Lagrangians. The first candidate is $\sigma^{[AB][CD]} = f^{EAB} f^{ECD} (= \delta^{AC} \delta^{BD} - \delta^{AD} \delta^{BC})$. This choice gives the term

$$\sigma^{[AB][CD]} c^A c^B \bar{c}^C \bar{c}^D = (\bar{c} \times c) \cdot (c \times c) = -2(\bar{c} \times c) \cdot (\bar{c} \times c),$$

and (2.10) becomes

$$\int Dc D\bar{c} (\bar{c} \times c) \cdot (\bar{c} \times c) e^{-\int dx \mathcal{L}_\alpha}. \quad (2.11)$$

From (2.8), the equality

$$\int d\zeta e^\zeta = 1 \quad (2.12)$$

holds. Therefore, as in the instanton case [15], (2.11) is derived from the nonvanishing partition function

$$\int Dc D\bar{c} e^{-\int dx \frac{K_1}{4} (i\bar{c} \times c)^2} e^{-\int dx \mathcal{L}_\alpha}, \quad (2.13)$$

where K_1 is a dimensionless constant.

Interaction with other fields is also possible. If we use $\sigma^{[AB][CD]} = B^E B^F (f^{EAC} f^{FBD} - f^{EBC} f^{FAD})$,² we obtain the term

$$\sigma^{[AB][CD]} c^A c^B \bar{c}^C \bar{c}^D = -2[B \cdot (c \times \bar{c})][B \cdot (c \times \bar{c})],$$

and (2.10) becomes

$$\int Dc D\bar{c} [B \cdot (c \times \bar{c})][B \cdot (c \times \bar{c})] e^{-\int dx \mathcal{L}_\alpha}. \quad (2.14)$$

Taking account of (2.12), we find (2.14) is derived from

$$\int Dc D\bar{c} e^{-\int dx K_2 B \cdot (\bar{c} \times c)} e^{-\int dx \mathcal{L}_\alpha}, \quad (2.15)$$

where K_2 is a dimensionless constant.

We can combine (2.13) and (2.15) in a BRS invariant form. Carrying out the BRS transformation

$$\delta_B A_\mu = D_\mu c, \quad \delta_B c = -\frac{g}{2} c \times c, \quad \delta_B \bar{c} = iB, \quad (2.16)$$

we obtain

$$\delta_B \left\{ \frac{K_1}{2} (i\bar{c} \times c)^2 + K_2 [B \cdot (\bar{c} \times c)] \right\} = (-iK_1 - gK_2) (B \times c) \cdot (\bar{c} \times c).$$

If we set $K_2 = -\frac{i}{g} K_1 = ig\alpha_2$, we get the BRS invariant effective Lagrangian

$$\mathcal{L}_{\text{eff}} = -\frac{\alpha_2}{2} (ig\bar{c} \times c)^2 + \alpha_2 B \cdot (ig\bar{c} \times c) = \frac{\alpha_2}{2} B^2 - \frac{\alpha_2}{2} \bar{B}^2, \quad (2.17)$$

where $\bar{B} = -B + ig\bar{c} \times c$, and α_2 is a new dimensionless constant.

² Instead of B , we can use A_μ . Examples are $F_{\mu\nu}$ and $\partial_\mu A_\mu$. However, using them, we cannot construct a Lagrangian which has mass dimension four (or lower than four) and has the off-shell BRS invariance.

Here we used the property (2.8) to derive the effective Lagrangian (2.17). In Appendix B, we derive it by using a source term.

Now we summarize the result. In the Landau gauge, when the configuration A_μ on the Gribov horizon contribute to the partition function, the FP operator has zero modes. If a pair of zero modes $(u_0(x), u_0^*(x))$ exists, the effective Lagrangian (2.17) is produced. From (2.1) and (2.17), we obtain the partition function

$$Z = Z_{\alpha=0}^{NL}$$

$$Z_{\alpha}^{NL} = \int D\mu e^{-\int dx (\mathcal{L}_{\text{inv}} + \mathcal{L}_{\alpha} + \mathcal{L}_{\text{eff}})} = \int D\mu e^{-\int dx (\mathcal{L}_{\text{inv}} + \mathcal{L}_{NL})}, \quad (2.18)$$

$$\mathcal{L}_{NL} = B \cdot \partial_\mu A_\mu + i\bar{c} \cdot \partial_\mu D_\mu c - \frac{\alpha_1}{2} B^2 - \frac{\alpha_2}{2} \bar{B}^2, \quad (2.19)$$

where $\alpha_1 = \alpha - \alpha_2$. Thus the Gribov horizon yields the Lagrangian in the nonlinear gauge \mathcal{L}_{NL} [16–18].

3 $\alpha \neq 0$ gauge

In the $\alpha \neq 0$ gauge, as $\partial_\mu A_\mu \neq 0$ and

$$\int dx i\bar{c} \cdot \partial_\mu D_\mu c = \int dx i(D_\mu \partial_\mu \bar{c}) \cdot c, \quad \partial_\mu D_\mu \neq D_\mu \partial_\mu,$$

the operator $\partial_\mu D_\mu$ is not hermitian. We assume that the operator $\partial_\mu D_\mu$ has a pair of zero modes (u_0, u_0^*) and a real single zero mode v_0 . Then c is expanded as

$$c(x) = \xi u_0(x) + \xi^\dagger u_0^*(x) + \zeta v_0 + \cdots, \quad (3.1)$$

where ξ, ξ^\dagger and ζ are independent Grassmann variables. Although the Lagrangian (2.2) does not contain ξ, ξ^\dagger and ζ , the measure Dc contains $d\xi d\xi^\dagger d\zeta$. Thus we find

$$\int Dc D\bar{c} e^{-\int dx \mathcal{L}_\alpha} = 0,$$

$$\int Dc D\bar{c} c^A c^B c^C e^{-\int dx \mathcal{L}_\alpha} \neq 0. \quad (3.2)$$

However (3.2) contradicts with the ghost number conservation. To avoid this problem, a pair of zero modes (\bar{u}_0, \bar{u}_0^*) and a real single zero mode \bar{v}_0 of the operator $D_\mu \partial_\mu$ must exist,³ and

³ Let us consider a square matrix \mathcal{D} , which is not necessarily hermitian. There are eigenvectors V_k which satisfy $\mathcal{D}V_k = \lambda_k V_k$. Since $\det(\mathcal{D} - \lambda E) = \det({}^t\mathcal{D} - \lambda E)$, ${}^t\mathcal{D}$ has the same eigenvalues as \mathcal{D} . Thus we have ${}^t\mathcal{D}U_l = \lambda_l U_l$. As tU_l satisfies ${}^tU_l \mathcal{D} = \lambda_l {}^tU_l$, these eigenvectors satisfy ${}^tU_l V_k = 0$ if $\lambda_l \neq \lambda_k$ [19]. In the present case, we assign $\mathcal{D} = \partial_\mu D_\mu$, ${}^t\mathcal{D} = D_\mu \partial_\mu$, $V_k = (u_k, u_k^*, v_k)$ and $U_l = (\bar{u}_l, \bar{u}_l^*, \bar{v}_l)$.

\bar{c} is expanded as

$$\bar{c}(x) = \bar{\xi}\bar{u}_0 + \bar{\xi}^\dagger\bar{u}_0^*(x) + \bar{\zeta}\bar{v}_0(x) + \cdots. \quad (3.3)$$

Since $\partial_\mu D_\mu \neq D_\mu \partial_\mu$, a zero-mode pair (\bar{u}_0, \bar{u}_0^*) is different from (u_0, u_0^*) , and $\bar{v}_0 \neq v_0$.

Now we consider the effect of the zero-mode pairs (u_0, u_0^*) and (\bar{u}_0, \bar{u}_0^*) . Since the Lagrangian does not contain $\xi, \xi^\dagger, \bar{\xi}$ and $\bar{\xi}^\dagger$, and the measure contains $d\xi d\xi^\dagger d\bar{\xi} d\bar{\xi}^\dagger$, to obtain a non-zero partition function, we must repeat the consideration in §2. Namely the zero-mode pairs give rise to the effective Lagrangian \mathcal{L}_{eff} , and the nonlinear gauge is realized.

Next we study the terms ζv_0 in (3.1) and $\bar{\zeta}\bar{v}_0$ in (3.3). The Lagrangian \mathcal{L}_{eff} has the term $ig\alpha_2 B \cdot (\bar{c} \times c)$. Although this term is necessary to ensure the BRS symmetry, as

$$B \cdot (\bar{c} \times c) = B \cdot \{\bar{\zeta}\zeta\bar{v}_0(x) \times v_0(x) + \cdots\}, \quad (3.4)$$

the partition function does not vanish even if $DcD\bar{c}$ contains $d\zeta d\bar{\zeta}$.

Thus, when $\alpha \neq 0$, the partition function changes from (2.1) to (2.18), if the FP operator $\partial_\mu D_\mu$ has a pair of zero modes. This result is unchanged even if this operator has a single zero mode.

4 Renormalization group flow of α

We return to the gauge $\alpha = 0$, and assume $\partial_\mu D_\mu$ has a single zero mode v_0 . Now $\partial_\mu D_\mu = D_\mu \partial_\mu$ holds, we must set $\bar{v}_0(x) = v_0(x)$ in (3.3), i.e.

$$c = \zeta v_0(x) + \cdots, \quad \bar{c} = \bar{\zeta} v_0(x) + \cdots.$$

Since $v_0(x) \times v_0(x) = 0$, $\bar{c} \times c$ and (3.4) do not contain $\bar{\zeta}\zeta$. Namely we cannot say that $Z_{\alpha=0}^{NL} \neq 0$ is guaranteed.

To evade this difficulty, we first construct the partition function $Z_\alpha^{NL} \neq 0$, and then take the limit $\alpha \rightarrow 0$, i.e. $\lim_{\alpha \rightarrow 0} Z_\alpha^{NL}$.

From the Lagrangian \mathcal{L}_{NL} , the equation of motion for B is

$$\partial_\mu A_\mu - \alpha B = -ig\alpha_2(\bar{c} \times c).$$

So, when $\alpha \rightarrow 0$, the term $-ig\alpha_2(\bar{c} \times c)$ must be taken into account. In this section, treating the interactions perturbatively at the one-loop level, we study the behavior of α .

In Appendix C, we derive the renormalization group (RG) equations

$$\mu \frac{\partial \alpha_1}{\partial \mu} = \frac{g^2 C_2(G)}{16\pi^2} \alpha_1 \left(\frac{13}{3} - \alpha_1 \right), \quad \mu \frac{\partial \alpha_2}{\partial \mu} = \frac{g^2 C_2(G)}{16\pi^2} \alpha_2 \left(\frac{13}{3} - \alpha_2 \right), \quad (4.1)$$

which coincide with the results in Refs. [20] and [21].⁴ We emphasize that the equation for α_1 does not contain α_2 , and vice versa. From (4.1), $\alpha = \alpha_1 + \alpha_2$ satisfies

$$\mu \frac{\partial \alpha}{\partial \mu} = \frac{g^2 C_2(G)}{16\pi^2} \left\{ \frac{13}{3} \alpha - \alpha^2 + 2(\alpha - \alpha_2) \alpha_2 \right\}. \quad (4.2)$$

When $|\alpha| \ll 1$, (4.2) becomes

$$\mu \frac{\partial \alpha}{\partial \mu} \simeq -\frac{g^2 C_2(G)}{8\pi^2} \alpha^2. \quad (4.3)$$

Therefore, when $\alpha_2 \neq 0$, α increases as μ decreases. The quartic ghost interaction makes $\alpha \neq 0$, and the situation in §3 realizes. Even if a single zero mode v_0 exists, the partition function does not vanish.

Eq.(4.1) shows that $(\alpha_1, \alpha_2) = (0, 0)$ is an infrared fixed point. Does this fact imply that the Landau gauge (1.1) is retrieved as $\mu \rightarrow 0$? Does the process in §2 repeat again? In the next section, we show such a trouble does not happen.

5 Ghost condensation

In Appendix B, we present the Lagrangian [18, 22]

$$\mathcal{L}_\varphi = -\frac{\alpha_1}{2} B^2 + B(\partial_\mu A_\mu + \varphi - w) + i\bar{c} \cdot (\partial_\mu D_\mu + g\varphi \times) c + \frac{\varphi^2}{2\alpha_2}. \quad (5.1)$$

This Lagrangian has the BRS invariance, if φ transforms as $\delta_B \varphi = g\varphi \times c$. Setting the constant $w = 0$, and performing the φ integration, we find \mathcal{L}_φ yields \mathcal{L}_{NL} . Namely, φ is an auxiliary field which represents $\alpha_2 \bar{B}$.

However, in a low energy region, φ is not an auxiliary field. In Ref. [22], we derived another RG equation for α_2 given by

$$\mu \frac{\partial}{\partial \mu} \alpha_2 = \frac{g^2 C_2(G)}{(4\pi)^2} (\beta_0 - 2\alpha_2) \alpha_2, \quad (5.2)$$

which is different from (4.1). Eq.(5.2) was derived by making the Wilsonian effective action for φ .⁵ We also showed that φ acquires the vacuum expectation value $\langle \varphi \rangle = \varphi_0$ under the

⁴ The parameters α_1 and α_2 in this article are related to the parameters in Refs. [20] and [21] as follows:

(1) after setting $\xi = 0, \zeta = \eta$ and $\alpha = \beta$, $\alpha_1 = (1 + \eta)\alpha$ and $\alpha_2 = -\eta\alpha$ in Ref. [20],
(2) $\alpha_1 = (1 - \xi)\lambda = \alpha' + \alpha/2$ and $\alpha_2 = \xi\lambda = \alpha/2$ in Ref. [21].

⁵ In Appendix C.2, we explain how to derive (5.2) from \mathcal{L}_{NL} .

energy scale

$$\mu_0 = \Lambda e^{-4\pi^2/(\alpha_2 g^2)}, \quad (5.3)$$

where Λ is a momentum cut-off. Ghost-antighost bound states and ghost condensation appear below μ_0 . We substitute $\varphi(x) = \varphi_0 + \varphi'(x)$ into (5.1), and choose the constant $w = \varphi_0$. This choice is necessary to maintain the BRS symmetry [23].⁶ Then (5.1) becomes

$$-\frac{\alpha_1}{2}B^2 + B(\partial_\mu A_\mu + \varphi') + i\bar{c} \cdot (\partial_\mu D_\mu + g\varphi' \times + g\varphi_0 \times)c. \quad (5.4)$$

Because of the dimensional transmutation [24], the parameter below μ_0 is not α_2 but φ_0 .

Contrary to α_2 , the gauge parameter α_1 remains in (5.4). As we explain in Appendix C.2, the RG equation (4.1) for α_1 persists, and $\alpha_1 = 0$ is an infrared fixed point. So, when $\mu \rightarrow 0$, (5.4) gives the gauge condition

$$\partial_\mu A_\mu + \varphi' \approx 0 \quad (5.5)$$

and the ghost Lagrangian

$$\begin{aligned} \int dx i\bar{c} \cdot (\partial_\mu D_\mu + g\varphi' \times)c &= \int dx i\bar{c} \cdot (D_\mu \partial_\mu)c \\ &= \int dx i(\partial_\mu D_\mu \bar{c}) \cdot c. \end{aligned}$$

As (5.5) means $\partial_\mu D_\mu \neq D_\mu \partial_\mu$, we assume $\partial_\mu D_\mu$ has a pair of zero modes (u_0, u_0^*) and a single zero mode v_0 , and $D_\mu \partial_\mu$ has zero modes (\bar{u}_0, \bar{u}_0^*) and \bar{v}_0 . Even if the measure $DcD\bar{c}$ contains $d\xi d\xi^\dagger d\zeta d\bar{\xi} d\bar{\xi}^\dagger d\bar{\zeta}$, because the term $i\bar{c} \cdot (g\varphi_0 \times c)$ in (5.4) has

$$-ig\varphi_0 \cdot \{\bar{\xi}\xi\bar{u}_0(x) \times u_0(x) + \bar{\xi}^\dagger \xi^\dagger \bar{u}_0^*(x) \times u_0^*(x) + \bar{\zeta}\zeta\bar{v}_0(x) \times v_0(x) + \dots\}, \quad (5.6)$$

the partition function does not vanish.

6 Summary

In the Landau gauge $\alpha = 0$, the FP operator $-\partial_\mu D_\mu$ has zero modes on the Gribov horizon. As the ghost c and the anti-ghost \bar{c} are Grassmann variables, it is natural to expect that these zero modes yield effective ghost interactions. We have shown the quartic ghost interaction is produced by a pair of zero modes. If we impose the BRS invariance, the Lagrangian in the nonlinear gauge is obtained. Thus the Landau gauge changes to the nonlinear gauge. In the $\alpha \neq 0$ gauge, the same result is obtained as well.

⁶ This point is explained in Appendix D. The anti-BRS symmetry and the global gauge symmetry are also discussed.

The effect of a single zero mode was also studied. Although there is no trouble in the $\alpha \neq 0$ gauge, the partition function Z may vanish in the $\alpha = 0$ gauge. We can avoid this problem by taking the limit $\alpha \rightarrow 0$.

Usually, when $\det \partial_\mu D_\mu = 0$ for some configuration A_μ , we can evade the $Z = 0$ problem by choosing another gauge (locally) [25]. In this paper, we have shown that such a configuration changes the gauge to the nonlinear gauge automatically.

The partition functions in the Landau gauge and the nonlinear gauge are equivalent perturbatively. In the nonlinear gauge, $(\alpha_1, \alpha_2) = (0, 0)$ is an infrared fixed point at the one-loop level. In this case, the Landau gauge is retrieved and the zero-mode problem appears again. However, this scenario is not true. The nonlinear gauge yield the ghost condensation below the energy scale μ_0 , and the zero-mode problem no longer happens.

A Examples of zero modes in the Coulomb gauge

In this appendix, choosing the gauge $\partial_j A_j = 0$, we study the eigenvalue equation

$$-\partial_j D_j u = -(\Delta + g A_j \times \partial_j) u = \lambda u \quad (\text{A1})$$

in three-dimensional space-time.

A.1 A pair of zero modes

If the eigenfunction has the form $u^A = e^{is} w^A$ with $g A_j \times (\partial_j w) = 0$, (A1) becomes

$$-i H^{AB} e^{is} w^B = (\Delta + \lambda) e^{is} w^A, \quad H^{AB} = g f^{ACB} A_j^C (\partial_j s). \quad (\text{A2})$$

Since H is a real antisymmetric 3×3 matrix, its eigenvalues are pure imaginary or 0, i.e.

$$H^{AB} w_+^B = i h(x) w_+^A, \quad H^{AB} w_-^B = -i h(x) w_-^A, \quad H^{AB} w_0^B = 0. \quad (\text{A3})$$

The last equation of (A3) means that the effect of A_j^C disappears and w_0 does not become a zero mode. From (A2) and (A3), we obtain

$$h(x) e^{\pm is} w_\pm^A = (\Delta + \lambda) e^{\pm is} w_\pm^A.$$

Thus we find the two functions $u_\pm = e^{\pm is} w_\pm$ become a zero-mode pair, if

$$h(x) u_\pm^A = \Delta u_\pm^A \quad (\text{A4})$$

holds.

To give concrete examples, let us choose the abelian configuration

$$A_i^A(\mathbf{x}) = a_i(\mathbf{x}) \delta^{A3}, \quad \partial_i a_i = 0. \quad (\text{A5})$$

A.1.1 Three-torus T^3

Gribov copies in the three-torus T^3 are studied in Ref. [5]. The constant configuration

$$a_j(x) = \frac{C_j}{gL}, \quad C_1 = 2\pi, \quad -2\pi < C_2 < 2\pi, \quad -2\pi < C_3 < 2\pi,$$

is on the first Gribov horizon, where L is the size of the torus. Setting $s = 2\pi x_1/L$, we find (A4) is satisfied by a zero-mode pair

$$u_{\pm} = e^{\pm i 2\pi x_1/L} \begin{pmatrix} 1 \\ \pm i \\ 0 \end{pmatrix}.$$

A.1.2 Axially symmetric configuration in \mathbb{R}^3

Next we consider the configuration

$$a_j(x) = \epsilon_{j3k} q(r) x_k, \tag{A6}$$

where (r, θ, ϕ) are the spherical coordinates. Using the angular momentum operator $\hat{L}_j = -i\epsilon_{jkl}x_k\partial_l$, we find

$$-\Delta = -\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{\hat{L}^2}{r^2}, \quad H^{AB} = gf^{A3B} q(r) i \hat{L}_3 s.$$

Then it is natural to set $e^{is} = e^{im\phi}$ and

$$w_+ = i^l R(r) \Theta_{lm}(\theta) \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix},$$

where l and m are integers, and

$$\Theta_{lm}(\theta) = \frac{(-1)^m}{\sqrt{2\pi}} \sqrt{\frac{(2l+1)(l-m)!}{2(l+m)!}} P_l^m(\cos \theta).$$

We note $e^{im\phi} \Theta_{lm}(\theta) = Y_{lm}(\theta, \phi)$ is the spherical harmonics which satisfies $Y_{lm}^* = (-1)^{-m} Y_{l,-m}$. Then (A4) becomes

$$\left[-\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial R(r)}{\partial r} + \frac{l(l+1)}{r^2} R(r) - gm q(r) R(r) \right] Y_{l,\pm m}(\theta, \phi) = 0. \tag{A7}$$

Now, following Henyey [26], we substitute the functions

$$R(r) = \frac{K r^\rho}{(r^2 + r_0^2)^\kappa}, \quad q(r) = \frac{d}{(r^2 + r_0^2)^\sigma} \tag{A8}$$

into (A7), where K, r_0, d, ρ, κ and σ are constants. Eq.(A7) is satisfied by

$$\sigma = 2, \quad \rho = l, \quad \kappa = l + \frac{1}{2}, \quad d = \frac{(2l+1)(2l+3)}{gm} r_0^2.$$

Thus we obtain the abelian configuration and the corresponding zero-mode pairs as

$$a_j = \frac{(2l+1)(2l+3)}{gm} \frac{r_0^2}{(r^2 + r_0^2)^2} \epsilon_{j3k} x_k,$$

$$u_{\pm} = i^l \frac{K r^l}{(r^2 + r_0^2)^{l+1/2}} Y_{l,\pm m}(\theta, \phi) \begin{pmatrix} 1 \\ \pm i \\ 0 \end{pmatrix}. \quad (l \geq 1, m = 1, 2, \dots, l) \quad (\text{A9})$$

In Ref. [26], the $l = 1$ case is presented explicitly.

A.2 A single zero mode

In Ref. [27], a single zero mode was found in an instanton background. Here we give an example in \mathbb{R}^3 . Generalizing (A5) and (A6), we choose the configuration

$$A_j^C(x) = \epsilon_{jCk} q(r) x_k. \quad (\text{A10})$$

Then (A1) becomes

$$-igq(r)\Xi^{AB}u^B = (\Delta + \lambda)u^A, \quad (\text{A11})$$

$$gf^{ACB}A_j^C\partial_j = igq(r)\Xi^{AB}, \quad \Xi^{AB} = f^{ACB}\hat{L}^C.$$

First we solve the equation

$$\Xi^{AB}u^B = i\alpha u^A, \quad (\text{A12})$$

where $i\alpha$ is an eigenvalue of Ξ . We substitute the expansion

$$u^A = \sum_{m=-l}^l R_{lm}^A(r) Y_{lm}(\theta, \phi),$$

and, for simplicity, choose $l = 1$. Then we find that the eigenvalues are $\alpha = 2, 1$ and -1 , and the numbers of eigenfunctions are 1, 3 and 5, respectively. We choose the real eigenfunctions

$u_\alpha^A = R_\alpha(r)w_\alpha^A(\theta, \phi)$, where $w_\alpha^A(\theta, \phi)$ are given by

$$\begin{aligned}\alpha = 2 : & \begin{pmatrix} Y_{11} - Y_{1,-1} \\ -i(Y_{11} + Y_{1,-1}) \\ -\sqrt{2}Y_{10} \end{pmatrix}, \\ \alpha = 1 : & \begin{pmatrix} \sqrt{2}Y_{10} \\ 0 \\ Y_{11} - Y_{1,-1} \end{pmatrix}, \begin{pmatrix} 0 \\ \sqrt{2}Y_{10} \\ -i(Y_{11} + Y_{1,-1}) \end{pmatrix}, \begin{pmatrix} i(Y_{11} + Y_{1,-1}) \\ Y_{11} - Y_{1,-1} \\ 0 \end{pmatrix}, \\ \alpha = -1 : & \begin{pmatrix} \sqrt{2}Y_{10} \\ 0 \\ -(Y_{11} - Y_{1,-1}) \end{pmatrix}, \begin{pmatrix} 0 \\ \sqrt{2}Y_{10} \\ i(Y_{11} + Y_{1,-1}) \end{pmatrix}, \\ & \begin{pmatrix} Y_{11} - Y_{1,-1} \\ i(Y_{11} + Y_{1,-1}) \\ 0 \end{pmatrix}, \begin{pmatrix} i(Y_{11} + Y_{1,-1}) \\ -(Y_{11} - Y_{1,-1}) \\ 0 \end{pmatrix}, \begin{pmatrix} Y_{11} - Y_{1,-1} \\ -i(Y_{11} + Y_{1,-1}) \\ 2\sqrt{2}Y_{10} \end{pmatrix}.\end{aligned}$$

Next we determine R_α . From (A11) with $\lambda = 0$ and (A12), R_α satisfies

$$-\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial R_\alpha(r)}{\partial r} + \frac{l(l+1)}{r^2} R_\alpha(r) + g\alpha q(r) R_\alpha(r) = 0. \quad (\text{A13})$$

As in the previous subsection, we substitute (A8) into (A13). Then we find

$$R_\alpha(r) = R(r) = \frac{Kr}{(r^2 + r_0^2)^{3/2}}, \quad q(r) = \frac{-15}{\alpha g} \frac{r_0^2}{(r^2 + r_0^2)^2}. \quad (\text{A14})$$

Two real zero modes are replaced by a pair of zero modes. So one real zero mode remains for each value of α .

B Derivation of the Lagrangians (2.19) and (5.1) by the use of "source"

In the instanton case, the fermion determinant does not vanish if fermion sources exist [13, 14]. Following this case, we introduce a field $\varphi(x)$, and replace $i\bar{c} \cdot \partial_\mu D_\mu c$ with

$$i\bar{c} \cdot [\partial_\mu D_\mu + g\varphi \times] c. \quad (\text{B1})$$

The eigenvalue equation is

$$-[\partial_\mu D_\mu + g\varphi \times] w_n = \Lambda_n w_n.$$

We treat the term $g\varphi \times$ as perturbation, and perform the expansion

$$w_n = w_n^{(0)} + w_n^{(1)} + \cdots, \quad \Lambda_n = \Lambda_n^{(0)} + \Lambda_n^{(1)} + \cdots,$$

where $\Lambda_n^{(0)} = \lambda_n$, and $w_n^{(0)} = u_n$ in (2.4) and $w_n^{(0)} = u_n^*$ in (2.5). Using the normalization $\int dx u_n^* \cdot u_n = 1$ and $\int dx u_n \cdot u_n = 0$, we obtain

$$\Lambda_n^{(1)} = g \int dx u_n^* \cdot (\varphi \times u_n)$$

for u_n and

$$\Lambda_n^{(1)*} = g \int dx u_n \cdot (\varphi \times u_n^*)$$

for u_n^* , where $f^{ABC} u_n^A \varphi^B u_n^C = 0$ has been used. Therefore, if $\partial_\mu D_\mu$ has a pair of zero modes (u_0, u_0^*) , (B1) gives rise to the determinant

$$\begin{aligned} \det[-\partial_\mu D_\mu - g\varphi \times] &= \prod_n |\Lambda_n|^{k_n} \approx |\Lambda_0^{(1)}|^2 \prod_{n \neq 0} |\Lambda_n|^{k_n} \\ &= \left| g \int dx u_0^* \cdot (\varphi \times u_0) \right|^2 \prod_{n \neq 0} |\Lambda_n|^{k_n}, \end{aligned} \quad (\text{B2})$$

where k_n is the number of eigenfunctions that have the eigenvalue Λ_n or Λ_n^* . Thus, although $\Lambda_0^{(0)} = \lambda_0 = 0$, $\Lambda_0^{(1)} \neq 0$ makes the partition function non-zero.

Since

$$\begin{aligned} \left| g \int dx u_0^* \cdot (\varphi \times u_0) \right|^2 &\propto \int d\xi d\bar{\xi} d\xi^\dagger d\bar{\xi}^\dagger \left[g \int dx \bar{\xi}^\dagger u_0^* \cdot (\varphi \times \xi u_0) \right] \\ &\times \left[g \int dy \bar{\xi} u_0 \cdot (\varphi \times \xi^\dagger u_0^*) \right], \end{aligned}$$

we find

$$Dc D\bar{c} \exp \left\{ -i \int dx \bar{c} \cdot (\partial_\mu D_\mu + g\varphi \times) c \right\} \quad (\text{B3})$$

gives the determinant (B2). To derive (2.19), we multiply (B3) by $\exp[-\int dx (\varphi + \alpha_2 B)^2 / (2\alpha_2)]$, and integrate with respect to φ :

$$D\varphi \exp \left\{ -i \int dx \bar{c} \cdot [\partial_\mu D_\mu + g\varphi \times] c - \int dx \left(\frac{\varphi^2}{2\alpha_2} + B\varphi + \frac{\alpha_2}{2} B^2 \right) \right\}.$$

After the φ integration, we obtain (2.19).

We note, to derive (5.1), (B3) must be multiplied by $\exp[-\int dx \{(\varphi - w + \alpha_2 B)^2 + 2w\varphi - w^2\} / (2\alpha_2)]$, where w is a constant determined later.

C Derivation of the RG equations (4.1) and (5.2)

In subsection C.1, using \mathcal{L}_{NL} , we derive the RG equation (4.1). In subsection C.2, the RG equation (5.2) is derived. The RG equation for α_1 under the scale μ_0 is discussed.

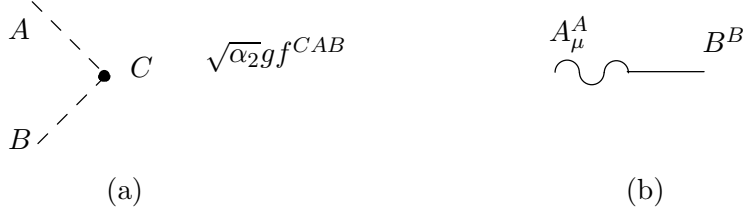


Fig. C1 The vertex and the propagator peculiar to \mathcal{L}_{NL} .

C.1 The Lagrangian (2.19) and the RG equations (4.1)

C.1.1 Equation for α_2

The Lagrangian \mathcal{L}_{NL} contains the quartic ghost interaction

$$-\frac{\alpha_2}{2}(ig\bar{c} \times c)^2.$$

We define the renormalization constant Z_4 by

$$(\alpha_2 g^2)_0 = Z_4 \tilde{Z}_3^{-2} \alpha_2 g^2, \quad (\text{C1})$$

where $\bar{c}_0 = \tilde{Z}_3^{1/2} \bar{c}$ and $c_0 = \tilde{Z}_3^{1/2} c$. First we consider the ghost self-energy. Although \mathcal{L}_{NL} gives additional one-loop diagrams, divergence of them cancels out. Thus we obtain, as usual, $\tilde{Z}_3 = 1 + \tilde{Z}_3^{(1)} + \dots$ with

$$\tilde{Z}_3^{(1)} = \frac{2g^2}{(4\pi)^2} (3 - \alpha) \frac{1}{4\varepsilon}, \quad (\text{C2})$$

where $\varepsilon = (4 - D)/2$, and $C_2(G) = 2$ is inserted. We note the gauge parameter in \mathcal{L}_{NL} is $\alpha = \alpha_1 + \alpha_2$.

Next we study Z_4 . Using the notation of Fig.C1, one-loop diagrams which contribute to Z_4 come from the diagrams in Figs.C2 and C3. However Fig.C2(b) does not yield divergence, and divergences of Figs.C2(c1)-(c3) cancel out. Furthermore some of the diagrams derived from Fig.C3 don't diverge.

Thus divergent diagrams are depicted in Fig.C4, and they give the constant $Z_4 = 1 + Z_{4a}^{(1)} + Z_{4b}^{(1)} + Z_{4c}^{(1)} + Z_{4d}^{(1)} + \dots$, where

$$\begin{aligned} Z_{4a}^{(1)} &= \frac{2g^2}{(4\pi)^2} (-\alpha_2) \frac{1}{\varepsilon}, & Z_{4b}^{(1)} &= \frac{2g^2}{(4\pi)^2} (-\alpha_2) \frac{1}{2\varepsilon}, \\ Z_{4c}^{(1)} &= \frac{2g^2}{(4\pi)^2} (\alpha_2) \frac{1}{\varepsilon}, & Z_{4d}^{(1)} &= \frac{2g^2}{(4\pi)^2} (-\alpha_1 - \alpha_2) \frac{1}{2\varepsilon}. \end{aligned} \quad (\text{C3})$$

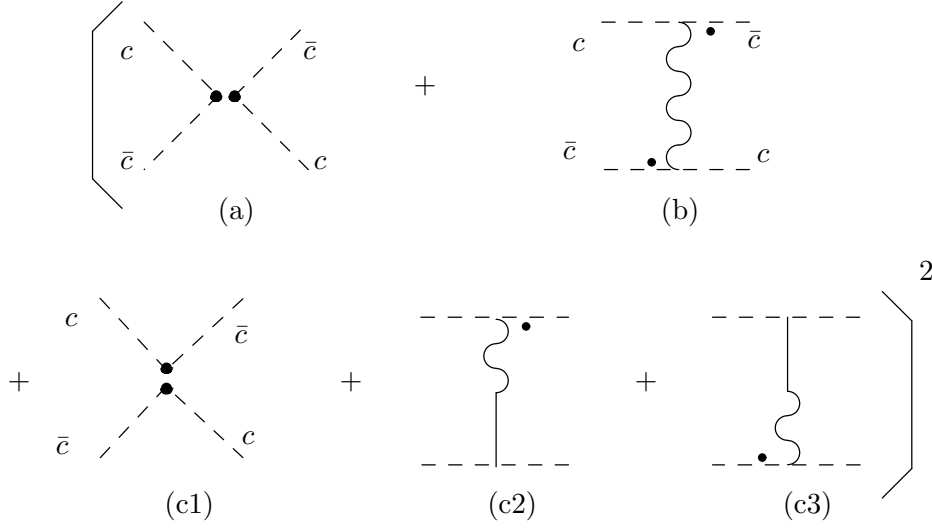


Fig. C2 The diagrams which contribute to one-loop correction for $(\bar{c} \times c)^2$ vertex.

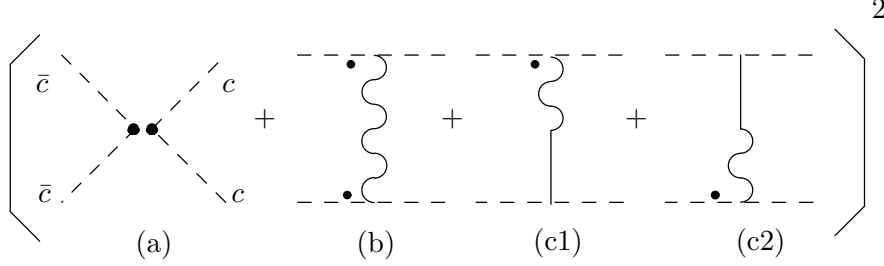


Fig. C3 The diagrams which contribute to one-loop correction for $(c \times c)(\bar{c} \times \bar{c})$ vertex.

Eq.(C1) leads to

$$\mu \frac{\partial \alpha_2 g^2}{\partial \mu} = -\frac{\mu}{Z_4 \tilde{Z}_3^{-2}} \frac{\partial Z_4 \tilde{Z}_3^{-2}}{\partial \mu} \alpha_2 g^2, \quad Z_4 \tilde{Z}_3^{-2} = 1 + Z_{4a}^{(1)} + Z_{4b}^{(1)} + Z_{4c}^{(1)} + Z_{4d}^{(1)} - 2\tilde{Z}_3^{(1)} + \dots \quad (\text{C4})$$

Then performing the replacement $g \rightarrow g\mu^{-\varepsilon}$ or $1/\varepsilon \rightarrow 2 \ln \Lambda/\mu$ in (C2) and (C3), and using the RG equation

$$\mu \frac{\partial g}{\partial \mu} = -\beta_0 \frac{g^3}{(4\pi)^2}, \quad \beta_0 = \frac{22}{3},$$

we obtain

$$\mu \frac{\partial}{\partial \mu} \alpha_2 = \frac{2g^2}{(4\pi)^2} \left(\frac{13}{3} - \alpha_2 \right) \alpha_2. \quad (\text{C5})$$

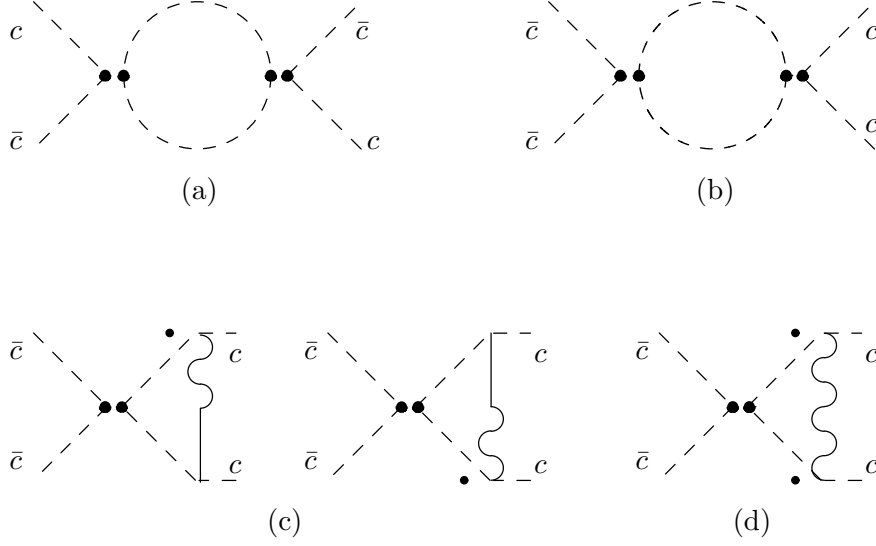


Fig. C4 The one-loop divergent diagrams for $(\bar{c} \times c)^2$ vertex.

C.1.2 Equation for α_1

Renormalization constants are defined as usual:

$$\begin{aligned} A_0^\mu &= \sqrt{Z_3} A^\mu, \quad Z_3 = 1 + Z_3^{(1)} + \dots, \quad B_0 = \sqrt{Z_B} B, \quad Z_B = 1 + Z_B^{(1)} + \dots, \\ (\alpha_j)_0 &= Z_{\alpha_j} \alpha_j, \quad Z_{\alpha_j} = 1 + Z_{\alpha_j}^{(1)} + \dots, \quad (j = 1, 2). \end{aligned} \quad (\text{C6})$$

Then \mathcal{L}_{NL} gives the counter terms

$$\frac{1}{2} (Z_B^{(1)} + Z_3^{(1)}) B \partial_\mu A^\mu, \quad \frac{1}{2} \left\{ (Z_B^{(1)} + Z_{\alpha_1}^{(1)}) \alpha_1 + (Z_B^{(1)} + Z_{\alpha_2}^{(1)}) \alpha_2 \right\} B^2.$$

The first counter term cancels the divergence of Fig.C5(a), and we obtain

$$Z_B^{(1)} + Z_3^{(1)} = \frac{2g^2}{(4\pi)^2} \frac{-\alpha_2}{\varepsilon}.$$

As the gauge parameter in \mathcal{L}_{NL} is α , the constant $Z_3^{(1)}$ is

$$Z_3^{(1)} = \frac{2g^2}{(4\pi)^2} \left(\frac{13}{3} - \alpha \right) \frac{1}{2\varepsilon} \quad (\text{C7})$$

as usual. Using these results, $Z_B^{(1)}$ becomes

$$Z_B^{(1)} = \frac{2g^2}{(4\pi)^2} \left(\alpha_1 - \alpha_2 - \frac{13}{3} \right) \frac{1}{2\varepsilon}. \quad (\text{C8})$$

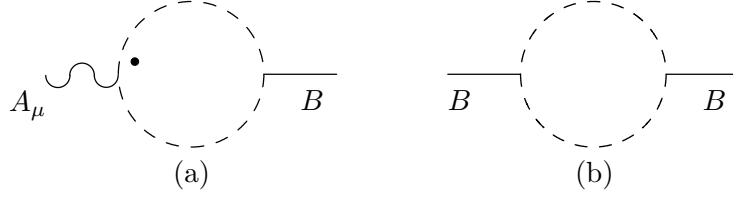


Fig. C5 The one-loop diagrams which contribute to the propagators for $A_\mu^A B^B$ and $B^A B^B$.

The divergence of Fig.C5(b) is canceled by the second counter term, i.e.

$$Z_B^{(1)}(\alpha_1 + \alpha_2) + Z_{\alpha_1}^{(1)}\alpha_1 + Z_{\alpha_2}^{(1)}\alpha_2 = \frac{2g^2}{(4\pi)^2} \frac{-\alpha_2^2}{\varepsilon}. \quad (\text{C9})$$

From (C5) and (C6),

$$\mu \frac{\partial}{\partial \mu} \alpha_2 = -\mu \frac{\partial Z_{\alpha_2}^{(1)}}{\partial \mu} \alpha_2 = \frac{2g^2}{(4\pi)^2} \left(\frac{13}{3} - \alpha_2 \right) \alpha_2$$

and

$$Z_{\alpha_2}^{(1)}\alpha_2 = \frac{2g^2}{(4\pi)^2} \left(\frac{13}{3} - \alpha_2 \right) \frac{\alpha_2}{2\varepsilon} \quad (\text{C10})$$

is derived. Substituting (C8) and (C10) into (C9), we obtain

$$Z_{\alpha_1}^{(1)}\alpha_1 = \frac{2g^2}{(4\pi)^2} \left(\frac{13}{3} - \alpha_1 \right) \frac{\alpha_1}{2\varepsilon}$$

and

$$\mu \frac{\partial}{\partial \mu} \alpha_1 = \frac{2g^2}{(4\pi)^2} \left(\frac{13}{3} - \alpha_1 \right) \alpha_1. \quad (\text{C11})$$

C.2 RG equations near μ_0 and under μ_0

C.2.1 Eq.(5.2)

The RG equation (5.2) is derived from the Lagrangian \mathcal{L}_φ [22]. To derive it from the Lagrangian \mathcal{L}_{NL} , we must replace (C4) with

$$Z_4 \tilde{Z}_3^{-2} \approx 1 + Z_{4a}^{(1)}. \quad (\text{C12})$$

Namely, in the region $\mu_0 < \mu < \Lambda$, the interaction between \bar{c} and c becomes strong, and Fig.C4(a) is the main contribution. In the limit $\mu \rightarrow \mu_0$, \bar{c} and c make bound states and ghost condensate.

C.2.2 RG equation for α_1

Near μ_0 , as we stated above, the Lagrangian (5.1) should be used. Under μ_0 , we must use the Lagrangian (5.4). In these Lagrangians, the gauge parameter for A_μ is not α but α_1 . Then, instead of (C7), we must use

$$Z_3^{(1)} = \frac{2g^2}{(4\pi)^2} \left(\frac{13}{3} - \alpha_1 \right) \frac{1}{2\varepsilon}.$$

Since the self-energies for BA_μ and BB don't have divergence now, $Z_{\alpha_1} = Z_B^{-1} = Z_3$ holds. Thus we have

$$\mu \frac{\partial}{\partial \mu} \alpha_1 = -\frac{\mu}{Z_3} \frac{\partial Z_3}{\partial \mu} \alpha_1 = \frac{2g^2}{(4\pi)^2} \left(\frac{13}{3} - \alpha_1 \right) \alpha_1.$$

That is, the RG equation for α_1 is unchanged.

D Symmetries of the Lagrangian \mathcal{L}_φ in (5.1)

D.1 BRS symmetry

It is easy to check that \mathcal{L}_φ is invariant under the BRS transformation

$$\delta_B A_\mu = D_\mu c, \quad \delta_B c = -\frac{g}{2} c \times c, \quad \delta_B \bar{c} = iB, \quad \delta_B B = 0, \quad \delta_B \varphi = g\varphi \times c.$$

The constant w is determined to conserve this symmetry. From the partition function

$$Z_\varphi = \int D\mu e^{-\int dx (\mathcal{L}_{inv} + \mathcal{L}_\varphi)},$$

we can derive the equation of motion for B as

$$\langle (-\alpha_1 B + \partial_\mu A_\mu + \varphi - w) \rangle = 0, \tag{D1}$$

where

$$\langle \Phi \rangle = \frac{1}{Z_\varphi} \int D\mu \Phi e^{-\int dx (\mathcal{L}_{inv} + \mathcal{L}_\varphi)}.$$

Since $D\mu$ and $\mathcal{L}_{inv} + \mathcal{L}_\varphi$ are invariant under the BRS transformation,

$$\langle \delta_B \Phi \rangle = 0 \tag{D2}$$

holds. We substitute $B = -i\delta_B \bar{c}$ and $\varphi(x) = \varphi_0 + \varphi'(x)$ into (D1), and use $\langle A_\mu \rangle = 0$, $\langle \varphi' \rangle = 0$. Then (D1) leads to $i\alpha_1 \langle \delta_B \bar{c} \rangle = w - \varphi_0$. The consistency with (D2) requires $w = \varphi_0$.

D.2 Anti-BRS symmetry

The anti-BRS transformation is given by

$$\bar{\delta}_B A_\mu = D_\mu \bar{c}, \quad \bar{\delta}_B \bar{c} = -\frac{g}{2} \bar{c} \times \bar{c}, \quad \bar{\delta}_B c = i\bar{B}, \quad \bar{\delta}_B B = gB \times \bar{c}, \quad \bar{\delta}_B \varphi = 0.$$

When $\varphi_0 \neq 0$, from the equation of motion for φ , $\langle \alpha_2 \bar{B} \rangle = \langle \varphi \rangle \neq 0$ holds. Therefore the anti-BRS symmetry is broken spontaneously, because

$$\langle \bar{\delta}_B c \rangle = \langle i\bar{B} \rangle \neq 0.$$

In addition, we must set $w = \varphi_0 \neq 0$ to maintain the BRS symmetry. As $\bar{\delta}_B \mathcal{L}_\varphi = -g(B \times \bar{c}) \cdot w$, the Lagrangian does not respect the anti-BRS symmetry.

D.3 Global gauge symmetry

Using the constant small parameter θ , the global gauge transformation is defined by $\delta_\theta \Phi = \theta \times \Phi$, where Φ represents all the fields in \mathcal{L}_φ . This symmetry breaks down just like the anti-BRS symmetry. In fact, $\varphi_0 \neq 0$ gives $\langle \delta_\theta \varphi \rangle = \theta \times \varphi_0$. and $w = \varphi_0$ brings $\delta_\theta \mathcal{L}_\varphi = -w \cdot (\theta \times B)$.

Next we study the partition function Z_φ . It transforms as $\delta_\theta Z_\varphi \propto \langle \delta_\theta \mathcal{L}_\varphi \rangle$. Using $B = -i\delta_B \bar{c}$ and (D2), we find

$$\delta_\theta Z_\varphi \propto -i(w \times \theta) \cdot \langle \delta_B \bar{c} \rangle = 0.$$

Namely, because of the BRS symmetry, Z_φ remains invariant under this symmetry.

In the same way, we can show that the breaking by w cannot be observed in any function $\langle \Psi(\Phi) \rangle$, if $\Psi(\Phi)$ is BRS-invariant. To show this, we consider the function

$$\langle \delta_\theta \mathcal{L}_\varphi \Psi(\Phi) \rangle, \tag{D3}$$

which appears in $\delta_\theta \langle \Psi(\Phi) \rangle$. Using $\delta_\theta \mathcal{L}_\varphi = -i(w \times \theta) \cdot \delta_B \bar{c}$ and $\delta_B \Psi(\Phi) = 0$, we find (D3) vanishes. Thus BRS invariant Green functions aren't broken by $\delta_\theta \mathcal{L}_\varphi$.

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